

## Computer-assisted teaching and learning of abstract photonics waveguide theory in undergraduate engineering technology programme

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**ABSTRACT:** Photonics and optical communications is the technology of choice for long distance, high data rate and high performance communication systems underlying the now ubiquitous Internet communications. Electromagnetic (E&M) theory is the essential foundation for students in this emerging technology field. Rigorous analytic and advanced mathematical skills are traditionally required to the understanding and applications of photonic device and waveguide designs. The use of standard industry software simulation packages to the teaching and learning is explored, and more importantly, for students to visualise the abstract photonics E&M concepts and theory. This teaching method improves teaching effectiveness of photonics E&M field and wave theories by helping the students better understand and visualise the mathematical complexities through commercial computer software tools. In addition to the better understanding of the theories, the students also gain the design experiences and skills using the industry standard software package and, therefore, better bridging the gap between the theory and real-world practices.

### INTRODUCTION

The vector property of electromagnetic (E&M) fields and waves is at the heart of optics and photonics E&M theories. It is also often a difficult knowledge point in an engineering and engineering technology curriculum. This is mainly due to the fact that the vector nature of E&M fields is extremely abstract [1-3]. In addition, the vector relationship between the fields and their responses in a medium is even harder to *see*. This non-intuitive nature of E&M fields has been for a long time one of the main obstacles for engineering students in their study of optics and photonics E&M theories. On the other hand, optics and photonics E&M theories are now becoming key components of core engineering knowledge as optics and photonics have become increasingly important in modern industries and electrical engineering practices. Optical fiber networks, for example, are the backbone of the Internet, data, voice and video communications. The need of proficient workers and design engineers in the field of photonic and opto-electronic technologies is growing and will continue to grow fast. Photonics E&M theory is essential to the engineering and technology students who want to develop a successful career in this field. Conventionally, however, teaching of E&M field and wave theory and photonics relies heavily, if not solely, on abstract and advanced mathematical analysis methods involving subjects such as partial differential equations (PDE), linear algebra, and vector calculus and analysis [1-3]. Students are required to solve wave equations analytically for optical fiber and waveguide designs and the procedure is strenuous even to mathematics majors. Accordingly, much of these waveguide theory and design are postponed into graduate study rather than appearing in the undergraduate curriculum. Moreover, since the boundary conditions that can entertain exact analytical solutions are only a few, the majority of practical designs still remains untouchable ever for traditional graduate engineering curricula.

Following the traditional teaching approach [1-3], an E&M field, represented typically by its electrical field (E field), is assumed to be linear polarised and the corresponding scalar PDEs are derived accordingly. In order to obtain the desired results including eigenmodes and the effective index of a waveguide, proper boundary conditions have to be applied. Mathematically, the Neumann continuity condition is usually used, and the scalar PDEs are solved by requiring the fields and their first derivatives to be continuous at the dielectric interfaces. This typical way of teaching has at least two shortcomings, one of which is conceptual and the other is practical. The conceptual shortcoming comes from the use of the mathematical Neumann boundary condition. The Neumann boundary condition requires that the field and its first derivative to be continuous across the boundary. However, at dielectric interfaces, for example, while the tangential fields are continuous, the normal component of the E field is not. Since the first derivative of the magnetic field (H field) is directly proportional to the E field, for example, the Neumann boundary condition does not hold obviously. How this mathematical boundary condition can be related to the physical boundary condition that is required for E&M fields is often a neglected topic in the traditional way of teaching. This again is due primarily to the lack of ability of students and sometimes teachers as well to *see* or visualise the vector relationship between E&M fields directly. On the application side, as practical designs grow in complexity, analytical techniques usually are far too complex and

cumbersome to serve as suitable and practical design tools. Standard practices in the industry use commercial and/or proprietary software simulation packages to address the design needs. It is, therefore, advantageous for engineering and technology students to get familiar with such simulation software packages and acquire such a highly desirable job skill. More importantly, the visualisation of the E&M fields and waves by the software's graphic user interfaces renders a direct experience and hands-on interaction for the students. An effective teaching method, accordingly, should combine the basic mathematical principles such as the Maxwell equations and wave equations with practical software tools that the students are more likely to use in their professional life.

In this article, the use of COMSOL®, a predominant industry PDE solver, in an engineering and technology senior level photonics course is discussed. Fundamental concepts such as transverse electric (TE) and transverse magnetic (TM) modes in planar waveguide designs are directly demonstrated in both analytical vector relations and the more intuitive graphs and pictures. This method of teaching enhances the effectiveness of teaching E&M and wave theories to engineering and technology undergraduate students through computer interactions that students can have with the mathematical complexities. In addition to the benefit of *seeing* the abstract relations, the students also acquire a practical design skill and, therefore, bridge better the gap between the theory and practice.

## BACKGROUND OF THE TEACHING MATERIAL: WAVE EQUATIONS AND BOUNDARY CONDITIONS

Wave equations derived from the Maxwell equations can take various field and/or potential representations. Typically, one uses either the electrical field (E field) or the magnetic field (H field) for solving waveguide eigenmode problems. The derivations of E field and H field wave equations are similar and the resulted wave equations are (1, 2, 3):

$$\nabla^2 \mathbf{E} + \frac{n^2 \omega^2}{c^2} \mathbf{E} = 0 \quad (1a)$$

$$\nabla^2 \mathbf{H} + \frac{n^2 \omega^2}{c^2} \mathbf{H} = 0 \quad (1b)$$

where  $n$  is the index of refraction,  $\omega$  is the frequency of light and  $c$  is the speed of light in vacuum. These are in general vector equations in three dimensional (3-D) space but most textbooks use the linear polarisation assumption to simplify Equation (1) into scalar wave equations. For planar waveguides, usually a one-dimensional (1-D) model is used as illustrated in Figure 1.

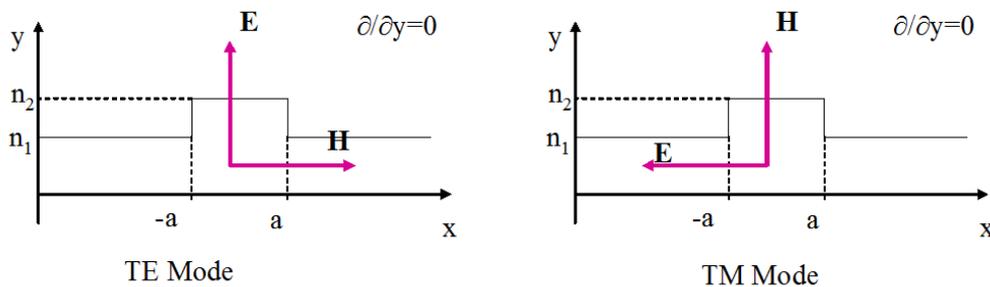


Figure 1: 1-D model of planar waveguide modes. Correspondingly, there are two types of eigenmodes, namely, TE modes, whose electrical field is parallel to the dielectric interface, and TM modes, whose electrical field is normal to the dielectric interface.

The 1-D eigenmode problem is given by

$$\frac{d^2}{dx^2} E + \frac{n^2 \omega^2}{c^2} E = \beta^2 E \quad (2)$$

where  $\beta$  is the effective propagation constant of the eigenmodes.

Traditionally, the teaching effort starting from here is to apply the above equation in each uniform domain of a structure and then apply the Neumann continuous boundary condition at each dielectric interface. This approach, however, can result in ambiguous explanations of the differences between TE and TM modes and easily cause confusion among students. This is especially true if a student wishes to correlate the mathematical Neumann continuous boundary condition here with the physical requirements of the dielectric interface. Recall that the Maxwell's E&M law requires that for a dielectric interface in lossless media the tangential components of both the E field and the H field are continuous while the normal component of the E field is not. The mathematical Neumann boundary condition, on the other hand, requires the field and its first derivative to be continuous.

In one typical traditional treatment,  $H_z$  is chosen as the field component to solve for TE modes, and  $E_z$  is chosen as the field component to solve for TM modes. However, in this case the first derivative of  $H_z$  or  $E_z$  (i.e.  $\partial H_z/\partial x$  and  $\partial E_z/\partial x$ ) does not by itself correspond to any field components. In fact,  $(\partial H_z/\partial x - \partial H_x/\partial z)$  corresponds to  $E_y$  and  $(\partial E_z/\partial x - \partial E_x/\partial z)$  corresponds to  $H_y$ . The continuity of  $\partial H_z/\partial x$  and  $\partial E_z/\partial x$  by itself at the dielectric interface is either an assumption or an approximation that cannot be traced back to the physical boundary conditions. Another popular treatment chooses  $E_y$  as the field component to solve for TE modes and  $E_x$  as the field component to solve for TM modes. In this case for TE modes, since  $\partial E_y/\partial x = j\omega\mu H_z$ ,  $E_y$  and  $\partial E_y/\partial x$  are, indeed, both continuous across the dielectric interfaces. However, for TM modes,  $\partial E_x/\partial x$  does not correspond to any field components and its continuity across the dielectric interfaces is again questionable. Even when  $H_y$  is correctly chosen as the field component to solve for TM modes, since  $\partial H_y/\partial x = -j\omega\epsilon E_z$ ,  $\partial H_y/\partial x$  is actually not continuous across the dielectric interfaces. In any of these cases of ambiguous treatments, a diligent student can easily get lost when trying to match the mathematical Neumann condition with the physical boundary conditions. A traditional way of teaching that correctly differentiates TE and TM modes starts with Equation (2) and applies different boundary conditions for TE and TM modes, and ends up with different sets of analytic relations for solving TE and TM eigenvalues  $\beta$  [3]. This approach quickly becomes intractable when several uniform regions are involved as it is usually the case in practical waveguide designs, notably, for example, for double-hetero-junction lasers [4]. For such multi-layer structure waveguide designs, commercial available PDE solver packages are commonly used in practice. COMSOL® is one of such packages that have gained increasing popularity in recent years. To a great benefit of engineering students, this software package also has a well designed graphical user interface with versatile graphical representations of the fields such as field contour plots and arrow plots. In the following sections, two simple, yet important, practical waveguide models that also help to visualise the E&M fields are demonstrated.

### VISUALISATION OF E&M FIELDS: OPTICAL FIBER MODES

A step-index optical fiber is introduced to students as the first model. This model is particularly simple in its geometry but is, indeed, very useful practically. The analytic approach for optical fiber model involves solving the 2-D eigenvalue problem in a cylindrical coordinate system and results in fiber mode profiles expressed in special functions. This approach is very abstract and the required mathematics is difficult for undergraduate engineering and technology students. Using COMSOL®, however, the vector wave equation is directly solved numerically by the finite element method (FEM) embedded in the software and the results can be exhibited by various field plots with easy and clear visualisation and interaction.

The vector wave equation is expressed in its general format,

$$\nabla \times \left( \frac{1}{n^2} \nabla \times \mathbf{H} \right) = \frac{\omega^2}{c^2} \mathbf{H} \quad (3)$$

Figure 2 shows the step-index optical fiber geometry and two of its degenerated fundamental modes with different polarisation. The arrow plots of the E field visually show the polarisation direction of each fundamental mode. To build this model, it only involves several simple steps of selecting the appropriate 2-D model (vector wave equation eigenvalue problem), drawing the fiber cross-section in its real length unit (um here) and input of the corresponding core and clad indices. Once the GUI is familiarised, it only takes our engineering and technology seniors half an hour to build and solve this model.

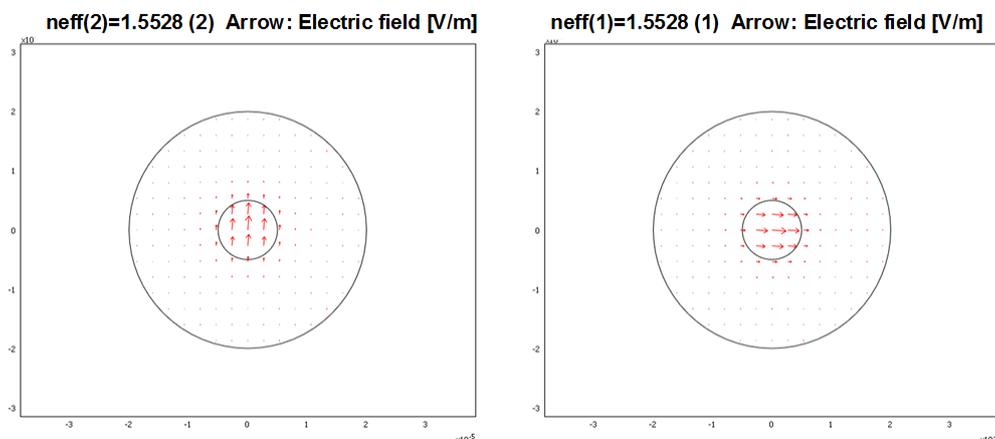


Figure 2: 2-D model of step-index optical fiber showing two of its polarisation degenerated fundamental modes. The arrow plots visually show the E field direction (polarisation) and the degenerated effective index.

The teaching results are very satisfying as the students are inspired by solving this complicated real world practical problem and visualising the results, or *seeing* the fields, for themselves. The students immediately reported that the

graphical representation helped them to understand the linear polarised modes in optical fiber. Indeed, for advanced students including graduate students, one can also visualise the  $z$ -component of the fields which will clearly indicate that the linear polarisation in optical fiber is only an approximation of the actual case.

The boundary condition for the circular dielectric interface for COMSOL® 2-D vector wave Equation (3) is also in the vector format:

$$\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = 0 \quad \text{and} \quad \mathbf{n} \times (\mathbf{E}_1 - \mathbf{E}_2) = 0 \quad (4)$$

which states that both tangential E field and tangential H field are continuous. This is the appropriate physical boundary condition, and is sufficient to uniquely determine the solution to the vector wave Equation (3).

### THEORY MEETS WITH PRACTICE: PLANAR WAVEGUIDE MODES BOUNDARY CONDITIONS

One of the objectives of introducing the COMSOL® photonics simulation package to our engineering and technology seniors is to equip them with a practical tool to design planar waveguides. Planar waveguides are used intensively in various integrated photonics devices, as well as semiconductor laser designs [4]. The proper PDE model here is a 1-D PDE eigenvalue problem. It is a standard practice to design planar waveguides, which are indeed a 2-D problem, by the method of effective index in two steps, first in the vertical ( $y$ ) direction and then in the in-plane ( $x$ ) direction [5]. The 1-D eigenvalue problem PDE equation that is applicable in COMSOL® reads:

$$\frac{d}{dx} \left( -c \frac{d}{dx} u \right) + au = -e_a \lambda^2 u \quad (5a)$$

for low loss 1-D waveguide models. The corresponding Neumann continuous boundary condition for Equation (5) reads:

$$\left( c \frac{du}{dx} \right)_1 - \left( c \frac{du}{dx} \right)_2 = 0 \quad (5b)$$

This boundary condition automatically takes into account both TE and TM cases provided that now Equation (2) only applies to TE modes, and TM wave equations take a slightly different format as:

$$\frac{d}{dx} \left( \frac{1}{n^2} \frac{d}{dx} H \right) + \frac{\omega^2}{c^2} H = \frac{1}{n^2} \beta^2 H \quad (6)$$

In each uniform region, Equation (2) and Equation (6) are exactly the same. Referring to Figure 1, both the E field of TE modes in Equation (2) and the H field of TM modes in Equation (6) are the  $y$ -component of the fields. Given the 1-D waveguide model, the index of refraction only varies along one spatial dimension, chosen here to be  $x$  direction. Since the medium is uniform in  $y$  direction, the derivatives respect to  $y$  vanishes. For TE modes, the  $z$  component of the E field is also zero.

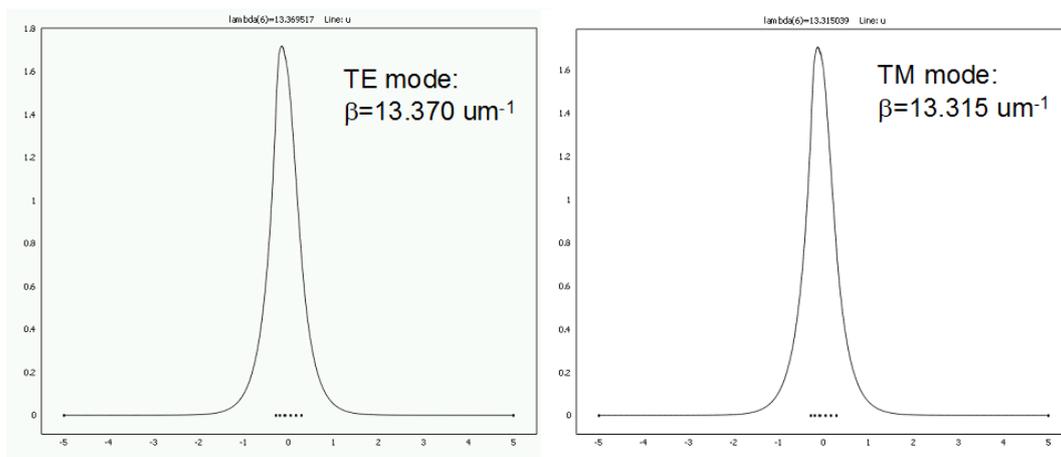


Figure 3: 1-D model of a multi-layer (5 different compositions, III-V semiconductors lattice matched to InP) planar waveguide model. Fundamental mode profiles and effective propagation constants for TE and TM modes respectively (student's course project work).

From the vector relationship between E field and H field, this implies that the only non-zero E field component is the y component, to which Equation (2) applies. Similarly for TM modes, the only non-zero H field component is also the y component, to which Equation (6) applies. Since all y components of the fields are tangential to the interface, they are all continuous as required physically. Their first derivatives, however, need to conform to Equation (5b) for the Neumann boundary condition to be physically valid. For TE modes Equation (5b) yields  $\left(\frac{dE}{dx}\right)_1 = \left(\frac{dE}{dx}\right)_2$ , where  $\left(\frac{dE}{dx}\right)_1$  is the first derivative of the E field respective to x evaluated at the boundary on the side of subdomain, one and  $\left(\frac{dE}{dx}\right)_2$  is the first derivative of the E field respective to x evaluated at the boundary on the side of subdomain two. Since  $dE/dx$  is proportional to the z component of H field,  $H_z$ , which is tangential to the interface, the Neumann boundary condition Equation (5b) correctly prescribes the physical boundary condition requirement for the E field of TE modes. For TM modes, Equation (5b) yields  $\left(\frac{1}{n^2} \frac{dH}{dx}\right)_1 = \left(\frac{1}{n^2} \frac{dH}{dx}\right)_2$ . Now that  $dH/dx$  is proportional to  $\epsilon_r E_z$ , and the z component of the E field,  $E_z$ , is continuous across the interface, therefore, Equation (5b) again correctly prescribes the physical boundary condition, recognising that  $\epsilon_r = n^2$ . Figure 3 shows a result of a multi-layer core planar waveguide model obtained by one of our engineering and technology seniors for their course project. By applying the COMSOL® software with careful considerations of boundary conditions, students expressed that they understand and appreciate much better the vector relationship among E&M fields. One student commented in his course evaluation that the best aspect of the course is *doing the COMSOL project and being able to learn the new software and how that software actually relates to what we were doing in the class.*

## DISCUSSION AND CONCLUSIONS

Commercial software package COMSOL® has been introduced in an engineering and technology senior course of laser electronics. It has been a great help to the instructor to help students to *see* the E&M fields and better understand the vector properties of E&M field like the polarisation state of light. The approach is effective also because it presents engineering and technology seniors with a practical tool for photonic and RF waveguide design.

The application of software to a course project, which in this case is to design an active semiconductor waveguide for diode lasers, deepens students' understanding of the related important topics such as TE and TM modes in planar waveguides. The results are satisfactory and the feedback from students is positive.

Combining with this department's previous efforts and experience using other popular commercial software packages such as MATLAB® in the classroom [6-8], it is believed that this effective approach is highly appropriate and advantageous for engineering and technology students to learn and, more importantly, in the case of photonic E&M theory, to visualise the vector nature of the E&M theory by combining the basic mathematical principles; for example, the Maxwell equations and wave equations, with practical software tools that they are more likely to use in their professional life.

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